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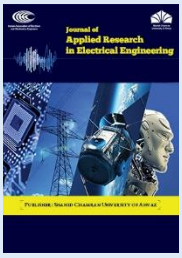

Research Article

Different Types of Distributed Optimal Leader-Follower Consensus Protocol Design for a Class of High-Order Multi-Agent Systems

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Abstract: Different types of optimal leader-follower consensus of high-order multi-agent systems (MAS) under fixed, connected, and directed communication topology are presented in this paper. The dynamics of each agent including the followers and their corresponding leader is a linear high order system. First, the Linear Quadratic Regulator (LQR) problem is discussed to achieve the optimal consensus for high-order linear MAS with a guaranteed predefined phase and gain margin. Then stochastic leader-follower consensus problem of MAS in the presence of the Gaussian noise is designed. To tackle these problems, a set of fixed distributed control laws for each follower agent is designed, based on algebraic graph theory. Simulation results indicate the effectiveness of the proposed method and display the consensus in both cases via distributed control laws.

Keywords: LQR Controller, LQG Controller, multi-agent systems, leader-follower consensus, fixed topology.

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1. Introduction

Simulation has become an indispensable tool for researchers to explore systems without having recourse to real experiments. Depending on the characteristics of the modeled system, the methods used to represent the system may vary. Multi-agent systems are, thus, often used to model and simulate complex systems. Control and optimization for multi-agent systems have drawn increasing interest in the past decade because they can be used to model practical examples in many fields, such as physical and information technology, economics, engineering, and social. In multi-agent systems, there are five types of problems: 1- Consensus 2- Containment 3- Tracking 4- Connectivity 5- Formation 6- Fault detection. In a consensus between agents, all agents try to achieve a constant value. This constant value is the average of the initial conditions of the agents. That is why it is called the average consensus. In such cases, the average consensus error is written and is tried to stably reduce this error to zero.

A. Bazaei et al. present a Linear Quadratic Gaussian (LQG) method to improve the tracking control performance of the constant-linear-velocity spiral reference [1]. This method is generally more robust compared to the inversion control technique. In [2] an optimal control strategy based on LQR controller is proposed for a group of agents to maintain formations while moving towards the destination. X. Li et al. investigate the distributed suboptimal LQR controller problem for continuous-time multi-agent systems [3]. In this reference, the distributed controllers are designed based on the system's topological structure such that each subsystem can use the information of its state and its neighbors. A. Iftar et al. study present a Linear Quadratic Gaussian/Loop Transfer Recovery (LQG/LTR) design methodology for decentralized control systems [4]. In [5] upper bounds on the norm of the multiplicative error matrix are calculated. They account for the interactions between the subsystems. F. Zhang et al. study the decentralized optimal control of discrete-time system with input delay and propose decentralized and centralized optimal controllers by the optimal tracking control use the LQR problem with delay. In [6] a Leveraging nonlinear model predictive control and a linear quadratic regulator (LQR) is developed as new methods for the active control of the spatial distribution of several such buoyancy actuated agents. Thus, in demonstrated a novel application of feedback control theory with LQR to an emerging real-world application in multi-agent systems. In [7] Lebao Li et al. present some common control algorithms of quadrotor unmanned aerial vehicle such as LQR, and analyse their merits and drawbacks. In addition, it is discussed that because of the limitations of single control algorithms, hybrid control schemes should be used to get the best performance. S. L. Nguyen et al. investigate the LQG game with a major player and a large number of minor players with mean field coupling [8].

In [9] A. M. Ferreira et al. present a self-tuning LQG/LTR method to design a controller for Thyristor Controlled Series Capacitors (TCSCs) with the objective of damping electromechanical power system oscillations. The optimal control problem in continuous space and time for collaborative multi-agent systems is investigated by W. Wiegerinck et al. [10]. In this method the agents in a stochastic environment have to distribute themselves over a number of targets. In [11] M. Nourian et al. study the large population leader-follower multi-agent systems where the agents have linear dynamics and are coupled via their quadratic cost functions. A mean field LQG stochastic control theory is presented for the large population dynamic game problem. Focusing on multi-robot navigation and collision avoidance applications, H. V. Henderson et al. propose a method to reduce the decentralized partially observed Markov decision process using the collection of decentralized LQG controllers for agents, thus, maximizing the joint performance of the team [12]. In [13] the problem of distributed linear-quadratic optimal control with agent-specified differential privacy requirements using the LQG problem has been studied. In [14] investigate the finite-time stability of a linear discrete time system with time-varying delays and its applications to the consensus problem of multi-agent system. In [15] using a control method similar to the LQR, the state-derivative and output-derivative feedbacks are derived for linear time-invariant systems. M. Rafiei sakhaei et al. provide a complete derivation for LQR optimal controllers and the optimal value function using basic principles from variational calculus [16]. As opposed to alternatives, the derivation does not rely on the Hamilton-Jacobi-Bellman equations, Pontryagin’s maximum principle, or the Euler-Lagrange equations. It provides a different perspective of how and why key quantities such as the adjoint variable and Riccati equation show up in optimal control computations and their connection to the optimal value function. In [17] a linear quadratic optimal hierarchical control problem is studied in which by using semi-groups the Kronecker product is specified for large-scale dynamical systems. The systems are modeled by an interconnected system under multi-scale information exchange networks and the cost is minimized using an algebraic approach. In [18] both continuous and discrete time consensus problems are studied for multi-agent systems with linear time-invariant agent dynamics over randomly switching topologies. In addition, the effect of Markovian switching topologies and random link failures on consensus are revealed. In [19] a problem of H_∞ consensus for nonlinear multi-agent systems with time-delay is investigated. A dynamic output feedback protocol is designed such that the multi-agent system reaches consensus in mean square with a prescribed H_∞ performance level. In [20] a class of consensus protocols are presented for directed networks of agents with general linear dynamics and synchronous intermittent information. In addition, some analytical results on consensus tracking of multi-agent systems with switching directed communication topologies are investigated. X. Wu et al study the distributed exponential consensus of delayed multi-agent systems with nonlinear dynamics under asynchronous switching [21]. L. D. Alvergue et al. investigate the output consensus control for continuous-time heterogeneous MASs, aimed at synchronizing all the agent’s output to the desired trajectory generated by a reference model [22]. The controller synthesis is based on H_∞ loop shaping and LQG/LTR methods, thereby the local optimality and stability robustness is ensured. In [23] the consensus control problems are reviewed along with the recent progress on stochastic MASs and the latest results on consensus analysis and protocol design issues for MASs are presented. A new pre-tuning multivariable PID (Proportional Integral Derivative) controllers' method for quadrotors has been investigated by R. Guardeño et al. [24]. A procedure based on LQR theory is proposed for attitude and altitude control. In this procedure a considerable simplification is supposed for the design problem, since only one pre-tuning parameter is used. The results show that the design method proposed for multivariable PID controllers is robust in the face of uncertainties and external disturbances acting at plant input. In [25] the consensus protocol for linear multi-agent systems with communication noises are studied. Each agent is allowed to have its own time-varying gain to attenuate the effect of communication noises. It is proved that if all noise-attenuation gains are infinitesimal of the same order, then the mean square leader-following consensus can be reached. Furthermore, the convergence rate of the multi-agent system and the steady-state performance and the transient performance are investigated. In [26] has been investigated the control of continuous-time linear Gaussian systems over additive white noise wireless fading channels subject to capacity constraints. It is shown that a separation principle holds between the design of the communication and the control subsystems, implying that the controller that would be optimal in the absence of the communication channel is also optimal for the problem of controlling the system over the communication channel. In [27] the problem of reduced order LQG optimization is investigated in a finite horizon, linear time-varying system setting. The authors in this reference provide first-order necessary conditions for local optimality in the parameter space using four coupled matrix differential equations.

This paper is the extension of reference [33] that was published by these authors and was selected as a top paper in AREE2021 conference. In this paper a method is proposed in which it is assumed that the communication topology between the leader and its neighbors depends upon bounded and time-invariant functions. To the best of our knowledge, this idea has not been investigated so far especially for agents with high order dynamics. In this study, the LQR and LQG methodology based on the Riccati equation and algebraic graph theory is employed to show the convergence of followers to leader optimally, while the communication link between the leader and its neighbors are fixed over time.

The paper is organized as follows: In Section 2 some notations on graph theory and Kronecker product are phrased. Then, in Section 3 system dynamics and basic mathematical foundation and assumptions are discussed. In Section 4, the proposed LQR and LQG controllers are designed. Section 5 presents the simulation results and comparing controllers, and in Section 6, the paper is concluded and the future research work is proposed.

2. Graph Theory and Kronecker Product

A directed graph is denoted as $G = (V, E)$ where $V = \{1,2, \dots, N\}$ is a finite and non-empty set of nodes (each node denotes the follower and there is N followers for $i = \{1,2, \dots, N\}$ and also $E \subset V \times V$ is a set of edges, each edge denotes an ordered pair of nodes). An edge (v_i, v_j) in an undirected graph shows that the agent i can access the information of the agent j and it means that the agent j is the neighbor of agent i. An adjacency matrix is $A_a = [a_{ij}] \in R^{N\times N}$. Moreover, it is assumed that $a_{ii} = 0$ for $i = \{1,2, \dots, N\}$. The set of neighbors of agent i is denoted by $N_i = \{j|(v_i, v_j) \in E\}$, Define the degree matrix as $D = diag\{d_1, \dots, d_N\}$ with $d_i = \sum_{j\in N_i} a_{ij}$. The symmetric Laplacian matrix corresponding to the directed graph G is defined as follows:

$$L = (D - A_a) \in R^{N\times N} \tag{1}$$

The leader agent is represented by vertex L and information is exchanged between the leader and the followers that are the neighbors of the leader [28]. A tool that is very useful in modeling and manipulating equations governing group motion is the Kronecker product $\otimes$ [29]. Kronecker product is also known as tensor product or direct product. Suppose C is the field of mixed numbers and $C^{m\times n}$ is a set of matrices containing m rows and n columns with mixed elements. For any matrices, $A = [a_{ij}] \in C^{m\times n}$ and $B \in C^{p\times q}$ their Kronecker product denoted as $A\otimes B \in C^{mp\times nq}$, is defined by:

$$A\otimes B = [a_{ij}B] = \begin{bmatrix} a_{11}B & \dots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \dots & a_{mn}B \end{bmatrix} \tag{2}$$

For example, if $\dot{x} = Ax_i$ represents the dynamics of a single agent, the dynamics of N identical agents can be represented as $\dot{x} = (I_N\otimes A)$. Another important case is when A is an $N \times N$ order matrix representing the manipulation of scalar data from N agents, and that the manipulation needs to be applied to each value of a vector of length n. In that case, the manipulation can be represented by concatenating the N vectors of length n into a single vector of length Nn, and multiplying it by $A\otimes I_n$. The following property of the Kronecker product:

$$(A\otimes B)(C\otimes D) = (AC)\otimes(BD) \tag{3}$$

can be proved when all matrix operations are well-defined [30]. In particular one of the most important properties of the Kronecker product is as follows:

$$A\otimes B = (A\otimes I_p)(I_n\otimes B) = (I_m\otimes B)(A\otimes I_q) \tag{4}$$

3. Problem Statement

Consider a MAS consisting of N followers and a leader. The dynamics of followers are linear n[th] order system is as follows:

$$\begin{cases} \dot{x}_i = Ax_i + Bu_i, \quad i = 1, \dots, N \\ y_i = Cx_i \end{cases} \tag{5}$$

where $x_i = [x_{i1}, \dots, x_{in}]^T \in R^n$ represents the states of agent i and also the term $u_i = [u_{i1}, \dots, u_{im}]^T \in R^m$ shows the control inputs of agent i. The matrices $A \in R^{n\times n}$ and $B \in R^{n\times m}$ are constant and represent the behavior of each follower [31]. The compact form of dynamics for N followers is as follows:

$$\begin{cases} \dot{X} = (I_N\otimes A)X + (I_N\otimes B)U \\ Y = (I_N\otimes C)X \end{cases} \tag{6}$$

where $X = [x_1, \dots, x_N]^T$, and $I_N = eye(N)$ and N is the number of agents.

The leader is the agent that is indexed by L, and the followers are the agents that are indexed by $i = 1, \dots, N$. The leader which is labelled with $i = L$ has linear n[th] order dynamics is as follows:

$$\begin{cases} \dot{x}_L = Ax_L + Bu_L \\ y_L = Cx_L \end{cases} \tag{7}$$

where $x_L = [x_{L1}, \dots, x_{Ln}]^T \in R^n$ shows the states of the leader. Obviously, the dynamics of the leader is independent of the others, has no input, and is an autonomous system, and is not affected by any of the followers. In this paper, the system matrices are considered the same for all followers and as well as the leader, because this case has practical background such as birds group, fishes school, etc. The compact form of leader dynamics is as follows:

$$\begin{cases} \dot{X}_L = (I_N\otimes A)X_L + (I_N\otimes B)U_L \\ Y_L = (I_N\otimes C)X_L \end{cases} \tag{8}$$

where $X_L = Ix_L$, $I = 1\otimes I_n$ and 1 is the N-vector of ones.

In this paper, the problem of designing distributed input law of U, which includes the inputs of all followers (i.e., $u_i, i = 1, \dots N$) to achieve leader-follower consensus is investigated, which is discussed in the following definition.

Definition 1: The leader-follower consensus of system (5) and (7) is said to be achieved, if for each follower $i \in \{1, \dots, N\}$, there is a local state feedback controller u_i of $\{x_i | j \in N_i\}$ such that the closed-loop system satisfies the following equation [32]:

$$\lim_{t\to\infty} \|x_i(t) - x_L(t)\| = 0, \qquad i = 1, \dots, N \tag{9}$$

for any initial condition $x_i(0)$, $i = 1, \dots, N$.

Consider the neighbourhood tracking error of the follower as follows:

$$e_i = \sum_{j\in N_i} a_{ij} (x_j - x_i) + b_i(x_L - x_i) \tag{10}$$

If $x_i \in R^n$, then the overall error form is:

$$E = ((L + B_b) \otimes I_n)(X_L - X) \tag{11}$$

where $I_n = eye(n)$ and n is the number of states, and $B_b = b_i \otimes I_N$ and $E = [e_1(t), \dots, e_N(t)]^T \in R^N$ is the vector of overall error. As the main contribution of this article, it is assumed that the communication topology between the leader and its neighbors is bounded and also depends on functions. The time derivative of this error along the compact form of dynamics of (6) and (8) is as follows:

$$\dot{E} = ((L + B_b) \otimes I_n)(\dot{X}_L - \dot{X}) \tag{12}$$

Consider the following change of variable:

$$\acute{A} = I_N \otimes \mathrm{A} \quad (13)$$

$$\acute{B} = I_N \otimes \mathrm{B} \quad (14)$$

Putting the equations of $\acute{A}$ and $\acute{B}$ in the overall error dynamics result that, finally the total error dynamics is as follows:

$$\dot{E} = ((L + B_b)\otimes I_n)\acute{A}\left((L + B_b)\otimes I_n\right)^{-1}E - ((L + B_b)\otimes I_n)\acute{B}U \quad (15)$$

Consider the following relationships:

$$A_n = ((L + B_b)\otimes I_n)\acute{A}\,((L + B_b)\otimes I_n))^{-1} \quad (16)$$

$$B_n = ((L + B_b)\otimes I_n)\acute{B} \quad (17)$$

Replacing A_n and B_n from the above equations in (15), $\dot{E}$ is obtained as shown in (18).

$$\dot{E} = A_n E - B_n U \quad (18)$$

4. Designing Leader-Follower Consensus Controller

In this section, the time-invariant overall leader-follower consensus controller of U which consists of the inputs of all followers (i.e., $u_i, i = 1, \dots N$) is designed. The LQG and LQR controllers are designed as follows.

The LQR cost function is as follows:

$$J = \int (E^T Q_i E + U^T R_i U)dt \quad (19)$$

Where matrix $Q_i$is positive semidefinite symmetric and matrix R_i is positive definite symmetric.

A hamiltonian function is derived as follows:

$$H = \frac{1}{2}E^T QE + \frac{1}{2}U^T RU + p^T(A_n E - B_n U) \quad (20)$$

By calculating $\frac{\partial H}{\partial E}$ co-state equation, $\dot{P}$ is obtained as follows:

$$\dot{P} = -QE - A_n^T p \quad (21)$$

Using the sufficient condition $\frac{\partial H}{\partial U} = 0$ and after some mathematical manipulations, U is obtained as follows:

$$U = R^{-1} B_n^T p \quad (22)$$

Replacing U in (18), derives the following relation:

$$\dot{E} = A_n E - B_n R^{-1} B_n^T p \quad (23)$$

The following Riccati equation for LQR is obtained:

$$\dot{K} + KA_n + KA_n^T + Q - KB_n R^{-1} B_n^T K = 0 \quad (24)$$

Now the distributed overall control law can be considered as (25).

$$\mathrm{U=KE} \quad (25)$$

The LQG cost function is as follows:

$$J = E(x_{(t_f)}^T H x_{(t_f)}) + \int (E^T Q_i E + U^T R_i U)dt) \quad (26)$$

The following Riccati equation for LQG is obtained:

$$-\dot{S} = A^T S + SA - SBR^{-1}BS + Q \quad (27)$$

The Riccati equation was calculated in the final condition $S_{t_f} = H$ and the system interest was obtained as follows:

$$K = R^{-1}B^T S \quad (28)$$

Now consider the distributed overall control law as follows:

$$\mathrm{U=KE} \quad (29)$$

5. Simulation Results

In this section, the proposed methodology is applied to the following system. Suppose that a connected, fixed, and directed communication network is depicted in Fig. 1 which is shown by the Laplacian matrix (L). The adjacency matrix of the above MAS is as follows:

$$A_a = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \quad (30)$$

$$L = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 2 & 0 & -1 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & -1 & 2 \end{bmatrix} \quad (31)$$

In this network, there are one leader and four followers. Bounded and functions for leader adjacency matrix are used, to keep the connectivity of the network. This means that if the network of agents is initially connected, it will stay connected as time passes.

The dynamics of each agent is a linear n[th] time-invariant high order system, which is relevant to the control of the Robot kinematics system. The attention that there are 4 followers which are shown by $i = 1, \dots, 4$ and the leader is an agent which is shown by $i = L$:

$$\begin{cases} \dot{x}_i = Ax_i + Bu_i + \xi_i \\ y_i = Cx_i + \eta_i \end{cases} \quad (32)$$

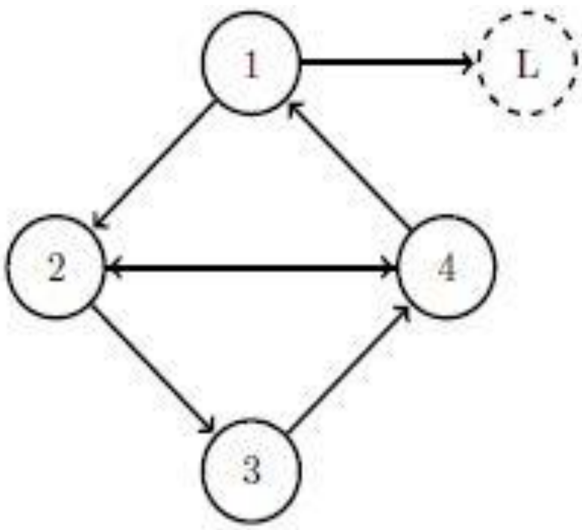

Fig. 1: The fixed and directed communication graph G for the MAS with one leader and four followers.

$$\begin{cases} \dot{x}_L = Ax_L + Bu_L \\ \quad y_L = Cx_L \end{cases} \tag{33}$$

In which the ξ_i is state noise and the η_i is output noise. The variance and mean noises are considered as follows:

$$\xi_i \simeq (1.1673 \times 10^{-6}, 1.0077 \times 10^{-8}) \tag{34}$$

$$\eta_i \simeq (0.0012, 0.0101) \tag{35}$$

Consider the system matrices as follows:

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \tag{36}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{37}$$

$$C = [1 \;\; 0] \tag{38}$$

It is easy to investigate that A and B are stabilizable, as well as in here the matrix A is stable.

In the simulation the initial conditions for the agents are considered as follows:

$x_1(0) = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, x_2(0) = \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix}, x_3(0) = \begin{bmatrix} 0.5 \\ 0.6 \end{bmatrix}, x_4(0) = \begin{bmatrix} 0.7 \\ 0.8 \end{bmatrix}$
$, x_L(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

5.1. LQR Controller

The LQR controller has been implemented on the above system. The results and the diagrams are shown in the following.

Under the proposed compact form of control law (25), the states of each follower track the states of the leader starting from any initial conditions. Figs. 2 and 3 show the difference between the first states of all followers and the leader, and the difference between the second states of all followers and the leader, respectively. As seen in Fig. 2, the first state of the follower is converged to the first of the leader. Also, Fig. 3 reveals that the second state of the followers converges asymptotically to zero. The control input of each follower is $u_i \in R^1$. Figs. 4 and 5 demonstrate the LQR controller input and the LQR Lagrange coefficient of all followers, respectively. Lagrange coefficients are the effect of states on the Hamiltonian, which tend to zero.

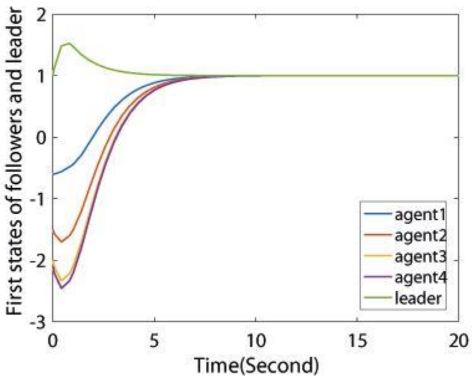

Fig. 2: The first states of followers and leader.

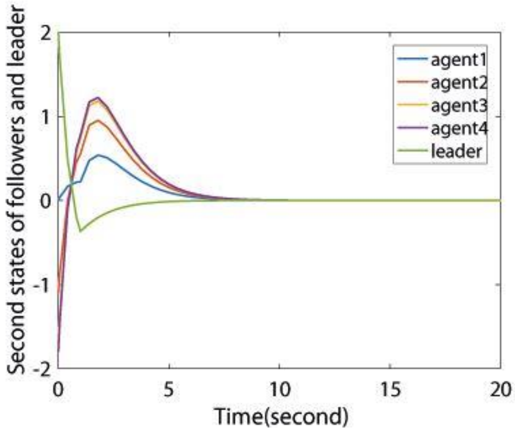

Fig. 3: The second states of the all followers and the leader.

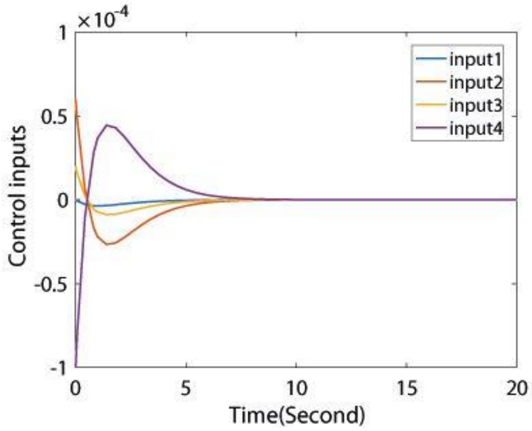

Fig. 4: The control inputs of the followers.

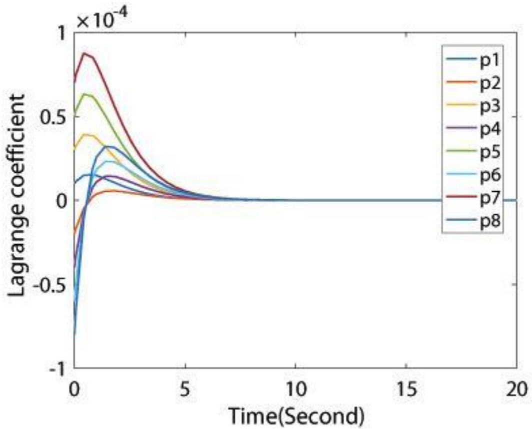

Fig. 5: The LQR Lagrange coefficient.

The consensus tracking errors of the first and the second states of the followers in the LQR controller design procedure are shown on Figs. 6 and 7, respectively. According to these figures, the consensus is well done. The results of the simulation show the hopeful efficiency of the proposed controller for MAS with fixed topology. The simulation results show the limitation of the control inputs, a convergence of the consensus error to zero, and the promising optimal performance with fixed topology.

5.2. LQG Controller

The LQG controller has been applied to the proposed system, the results and the diagrams are shown as follows. In Figs. 8 and 9 the first states of all followers and the leader and the second states of all followers and the leader is shown, respectively.

According to the figure, the first states of the followers lead to the first states of the leader. According to the figure, the second states of the followers proceed to the second states of the leader too. The states of each follower track the states of the leader starting from any initial conditions. It has been shown that all states of followers track the states of leader according to the proposed methodology. In the following figure, the LQG controller input is depicted as. In the following figure, the LQG Lagrange coefficients of all followers are displayed.

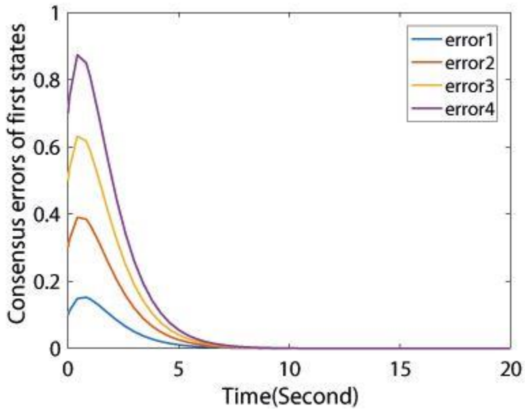

Fig. 6: The consensus errors of the first states of followers.

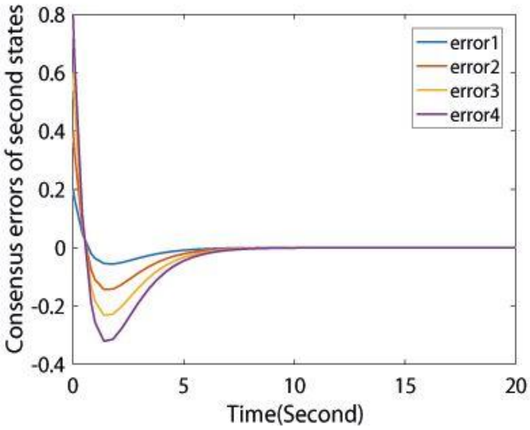

Fig. 7: The consensus errors of the second states of followers.

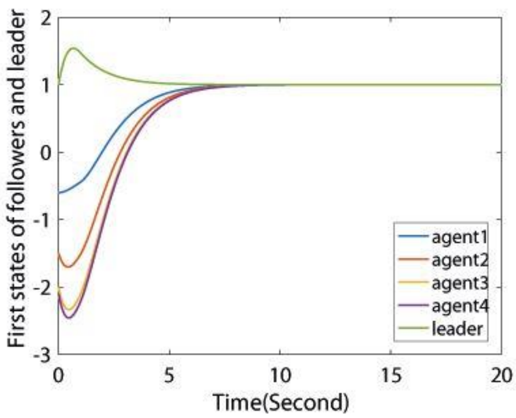

Fig. 8: The first states of followers and the leader.

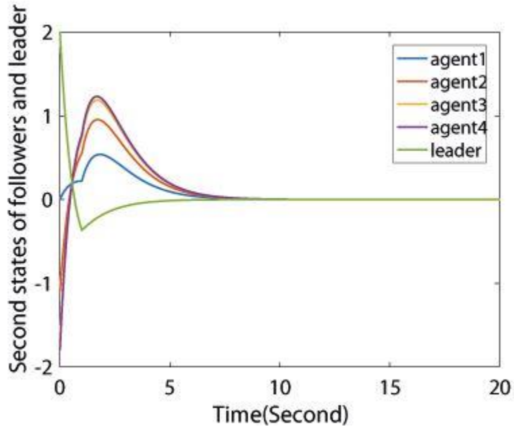

Fig. 9: The second states of the all followers and the leader.

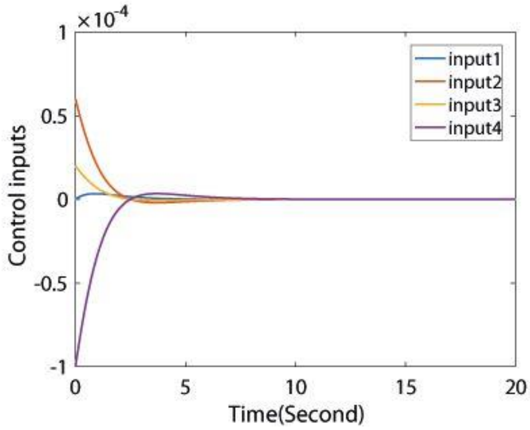

Fig. 10: The control inputs of the followers.

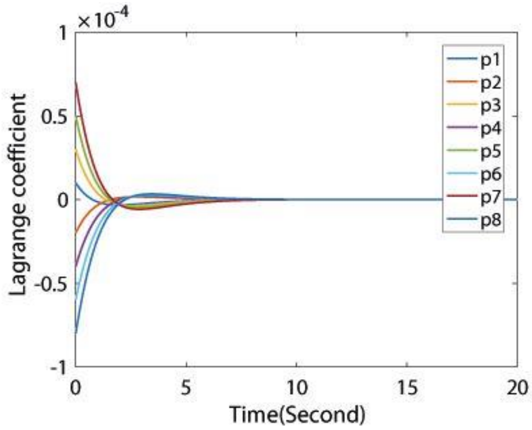

Fig. 11: The LQG Lagrange coefficient.

In the LQG controller design procedure, the consensus tracking errors of all states of the followers are shown as below. According to the figures, the consensus is well done. The results of the simulation show the hopeful performance of the proposed controller for the MAS with fixed topology and the limitation of the Control inputs, the convergence of the Consensus error to zero, and the optimal performance.

5.3. Comparison of LQR and LQG Tracking Error

The LQR and LQG tracking error is compared based on the Integral Square Error criterion in the following table. According to the table it is observed that convergence speed of the LQR methodology implemented is more than the LQG methodology.

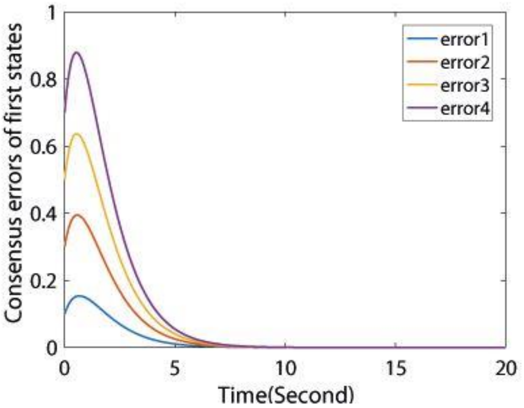

Fig. 12: The consensus errors of the first states of followers.

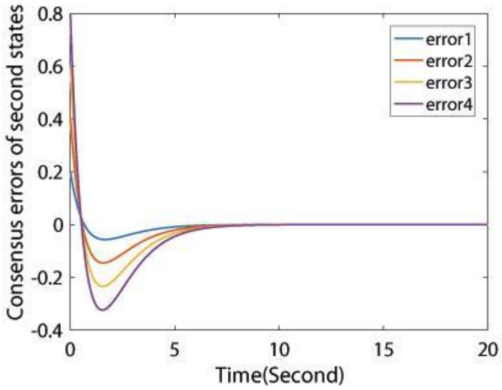

Fig. 13: The consensus errors of the second states of followers.

Table 1: ISE for LQR and LQG.

ISE	LQR	LQG
e1	0.1725	0.4808
e2	0.0682	0.1573
e3	1.1215	3.1002
e4	0.3194	0.7639
e5	2.9018	8.0021
e6	0.7634	1.8476
e7	5.5135	15.1869
e8	1.4003	3.4084

6. Conclusions

In this paper, the distributed leader-follower consensus of MAS is derived based on LQR and LQG under fixed, directed topology. The dynamics of each agent was a linear high order system. The distributed optimal controller is designated to reach a consensus for both the deterministic and stochastic MAS. The convergence of consensus error to zero in presence of the Gaussian noise is guaranteed. The prescribed gain and phase margin are achieved in our approach. The simulation results also indicated the promising efficiency of the presented methodology in stochastic and deterministic cases. Future research should focus on the formation of stochastic systems and the consensus of nonlinear MAS.

CRediT Authorship Contribution Statement

Farideh Azadmanesh: Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Roles/Writing - original draft, Writing - review & editing**. Reza Ghasemi:** Formal analysis, Methodology, Project administration, Supervision, Validation,

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. The ethical issues; including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, redundancy has been completely observed by the authors.

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